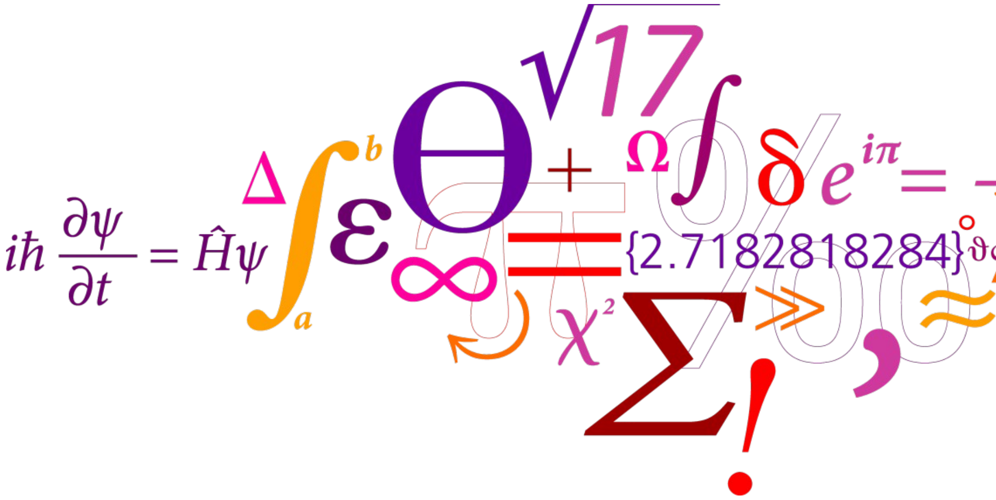
11. High Voltage Lifter

Presented by: Suzanne Zamany Andersen

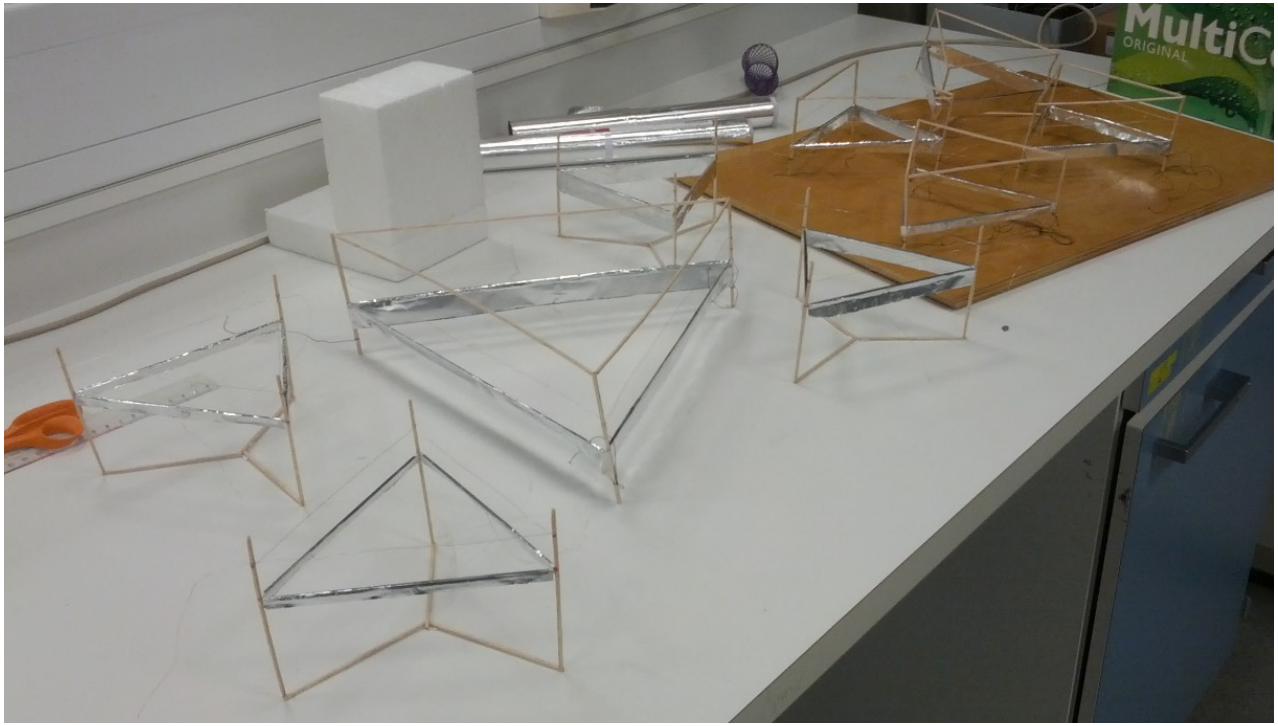
Construct the most powerful lifter possible with a surface area below 0.1 m^2



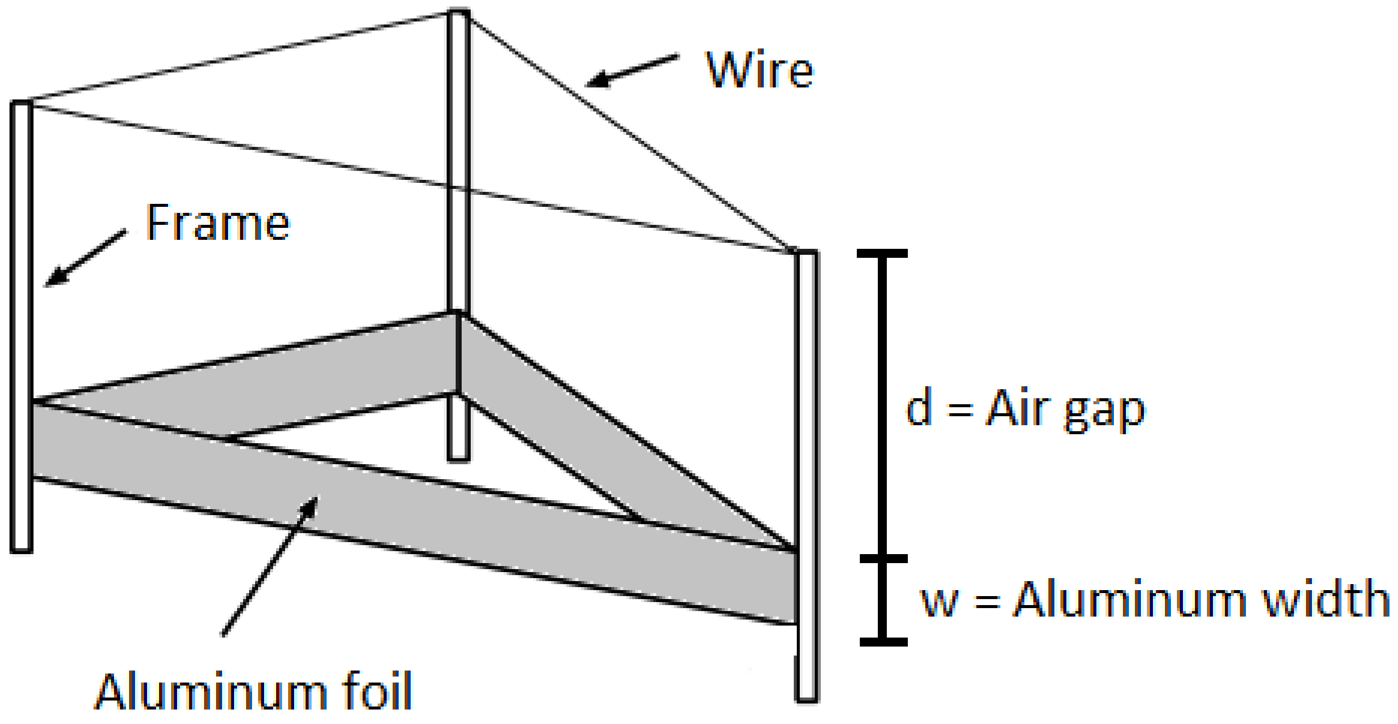
DTU Physics
Department of Physics

Problem description

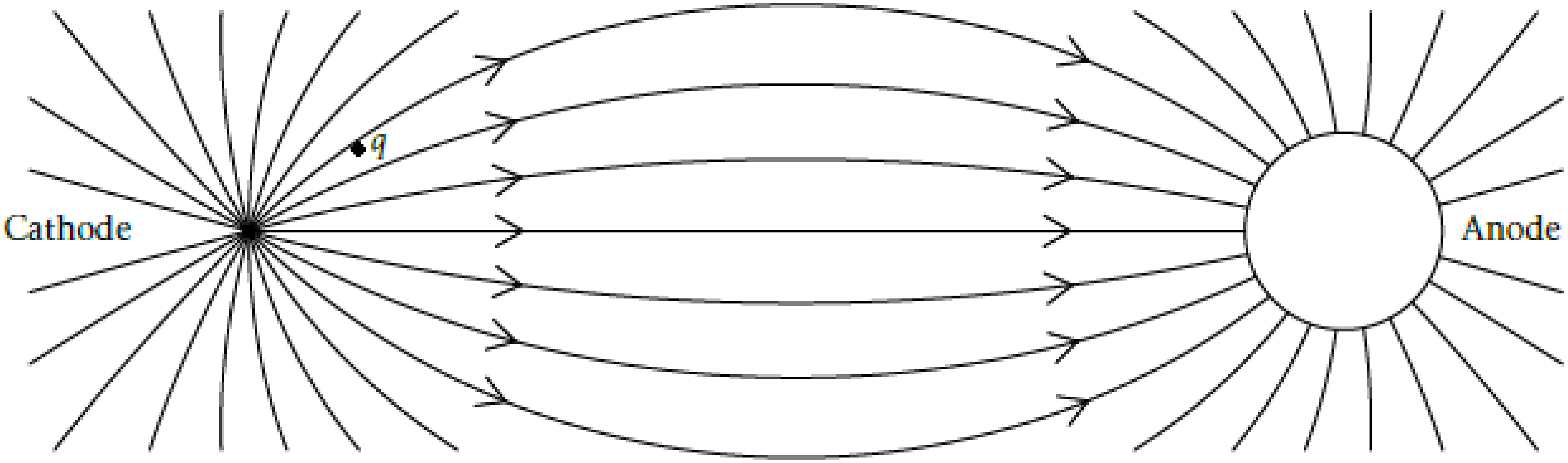
- Powerful=can lift the most weight, including its own weight.



Anatomy of lifter



Theory



[5] Nick Andersen, Kasper Larsen, ”The electrostatic levitation unit”, Technical university of Denmark, FYS, special project, 10064, 2008.

Theory: No applied voltage

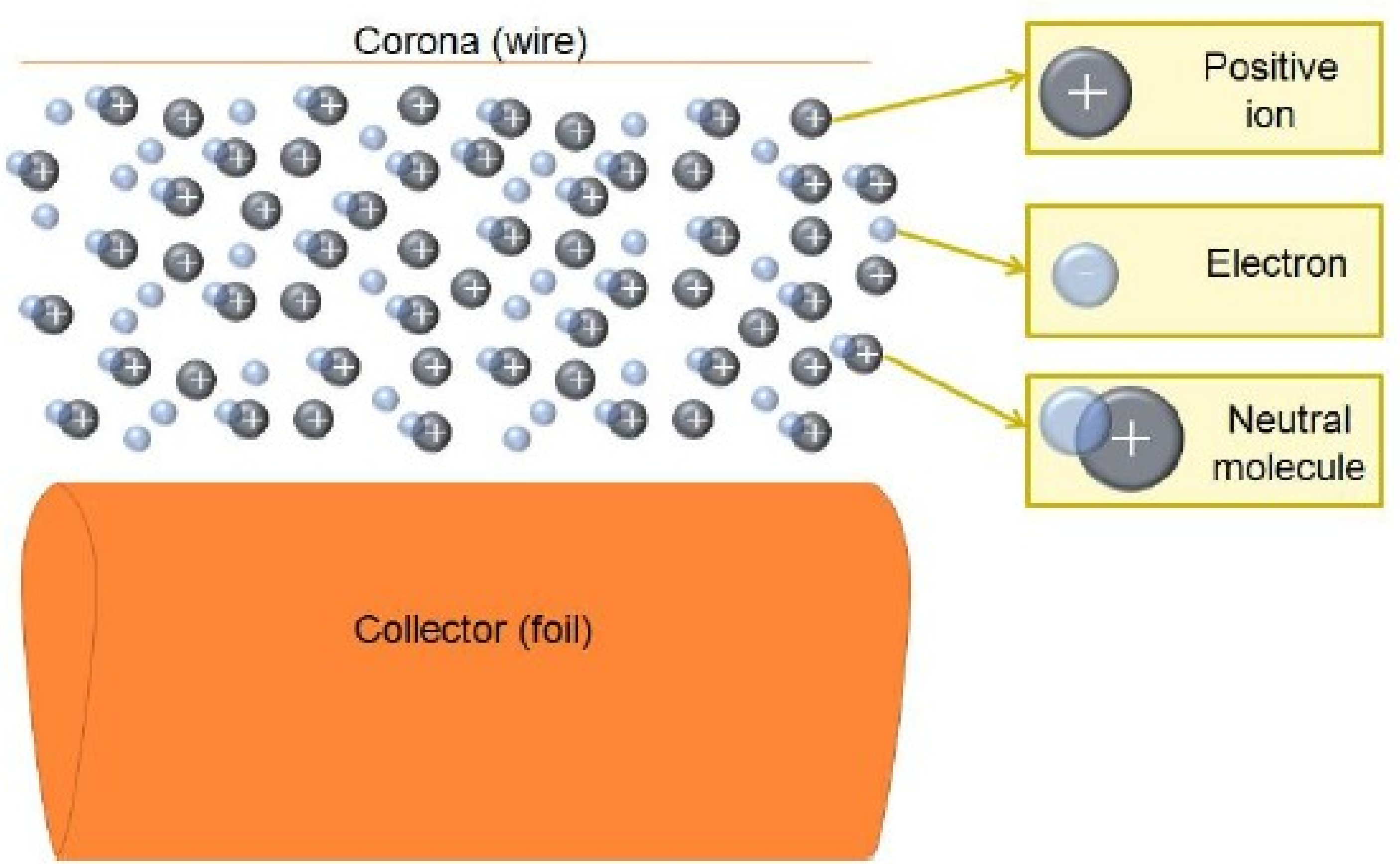


Figure from: [4] Clemens Wan, ”Electro-Hydrodynamic (EHD) thruster analysis and optimization”, The cooper union for the advancement of science an art, 2009.

Theory: Applying a high voltage

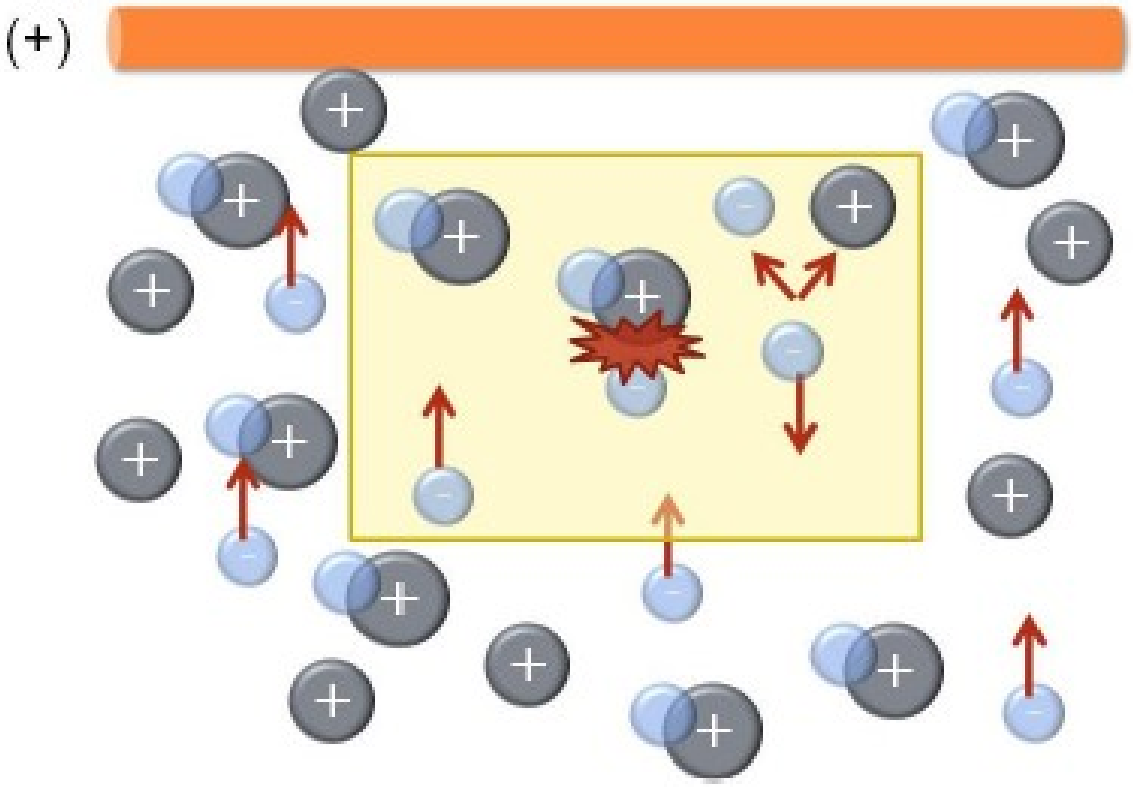


Figure from: [4] Clemens Wan, ”Electro-Hydrodynamic (EHD) thruster analysis and optimization”, The cooper union for the advancement of science an art, 2009.

Theory: EHD flow

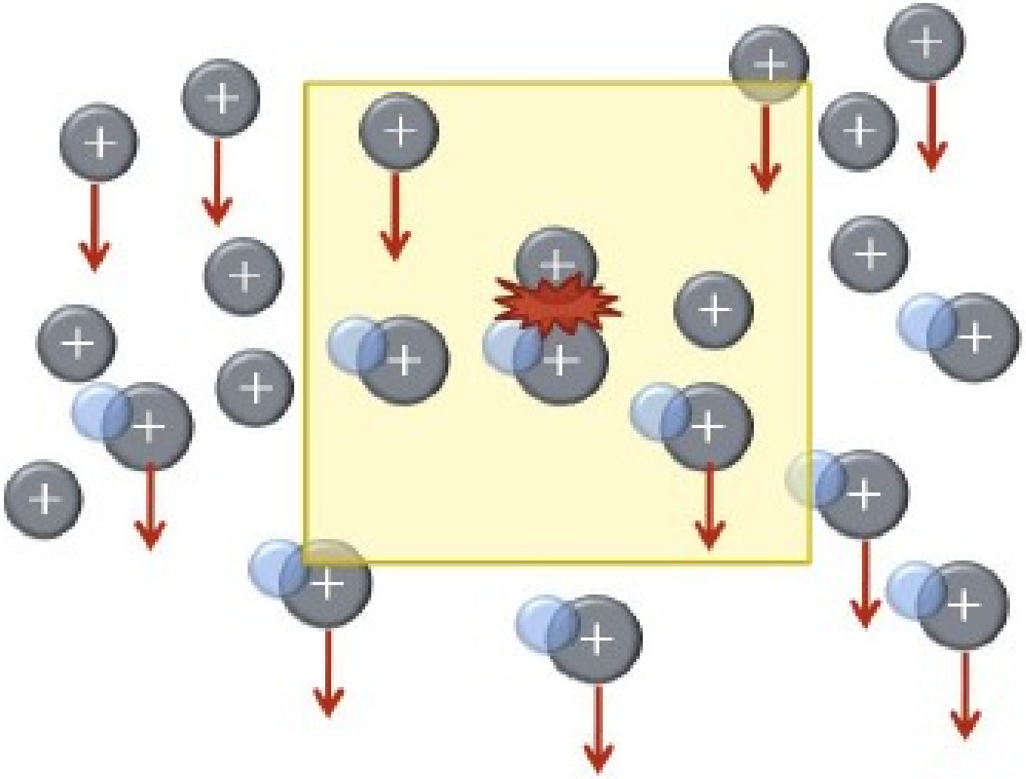


Figure from: [4] Clemens Wan, ”Electro-Hydrodynamic (EHD) thruster analysis and optimization”, The cooper union for the advancement of science an art, 2009.

Theory: Corona Inception Voltage

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- Peeks law for calculating Corono Inception Voltage (CIV) for two wire with equal radius:

$$CIV = E_i \cdot m_v \cdot r \cdot \ln\left(\frac{d}{r}\right)$$

$$E_i = E_0 \cdot \delta \cdot \left(1 + \frac{\gamma}{\sqrt{\delta \cdot r}}\right)$$

- E_0: electric field strength neccessary to break down air ($E_0 \approx 3 \cdot 10^6$ V/m)
- δ: Air density factor (At STP $\delta = 1$)
- γ: Peeks value given as $\gamma = 0.0301\sqrt{\mathrm{m}}$
- m_v: irregularity factor of wires. For smooth wires $m_v = 1$.

Theory: Current-voltage characteristic

-
  - Slightly changed Poisson equation:

$$\nabla \cdot E = \frac{1}{\varepsilon_0} \cdot (\rho_i + \rho_\sigma)$$

  - ρ_i is the ionic space charge density
  - ρ_σ is the space charge density of dispersive phase.

$$\rho_\sigma = \sigma \cdot \varepsilon_0 \cdot E$$

  - Cylindrical coor.: $\frac{1}{r} \cdot \frac{d(rE)}{dr} = \frac{\rho_i}{\varepsilon_0} + \sigma \cdot E$
  - Solution:

$$E = E_{0\sigma} \cdot r_0 \cdot \frac{e^{\sigma r}}{r} + \frac{\rho_i}{\varepsilon_0 \sigma^2}\left(\frac{e^{\sigma r}}{r} - \frac{1}{r} - \sigma\right)$$

  - $E_{0\sigma}$ is the electric field intensity at surface of coronating electrode

Theory: Current-voltage characteristic

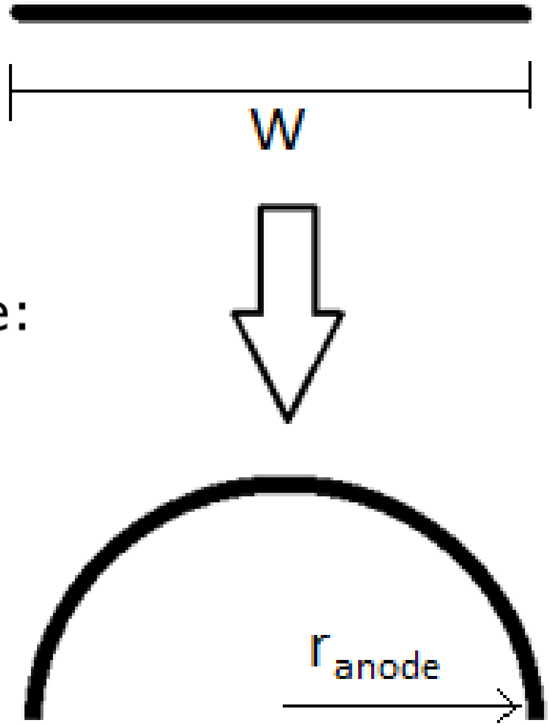
- A formatted Townsend equation is used for wire-plate capacitor setup. (see appendix A)

$$I = \frac{2\cdot\pi\cdot\varepsilon_0\cdot\kappa\cdot L}{d^2\cdot\ln\left(\frac{\frac{W\cdot\pi}{2}e^{\frac{\pi\cdot d}{W}}}{r}\right)}\cdot V(V-CIV)$$

- Since our anode is not a plate, we estimate W to be the top half of a circle:

$$W = \frac{1}{2}\cdot 2\cdot\pi\cdot r_{an}$$

$$I = \frac{2\cdot\pi\cdot\varepsilon_0\cdot\kappa\cdot L}{d^2\cdot\ln\left(\frac{\pi^2\cdot r}{2\cdot r_{an}}\cdot e^{d/r_{an}}\right)}\cdot V(V-CIV)$$



Theory: Generated force

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- We have a charge distribution function between the electrodes given as:

$$q = \frac{I \cdot d}{v_d}$$

$$v_d = \kappa \cdot E = \frac{\kappa \cdot V}{d}$$

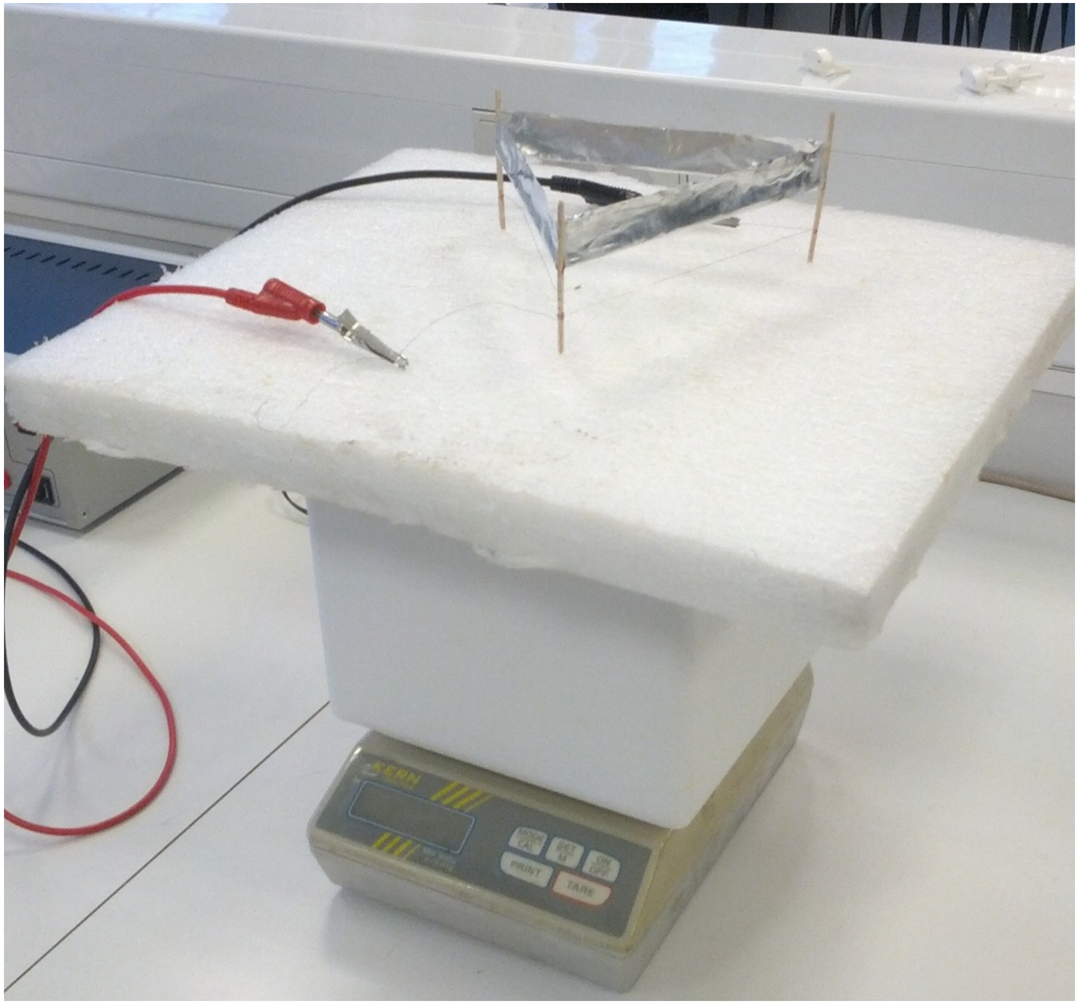
$$F = q \cdot E = \frac{q \cdot V}{d} = \left(\frac{I \cdot d}{v_d}\right) \cdot \left(\frac{V}{d}\right) = \left(\frac{I \cdot d}{\kappa \cdot V/_{d}}\right) \cdot \left(\frac{V}{d}\right) = \frac{I \cdot d}{\kappa}$$

$$F = \frac{2 \cdot \pi \cdot \varepsilon_0 \cdot L}{d \cdot \ln\left(\frac{\pi^2 \cdot r}{2 \cdot r_{an}} \cdot e^{d/r_{an}}\right)} \cdot V(V - CIV)$$

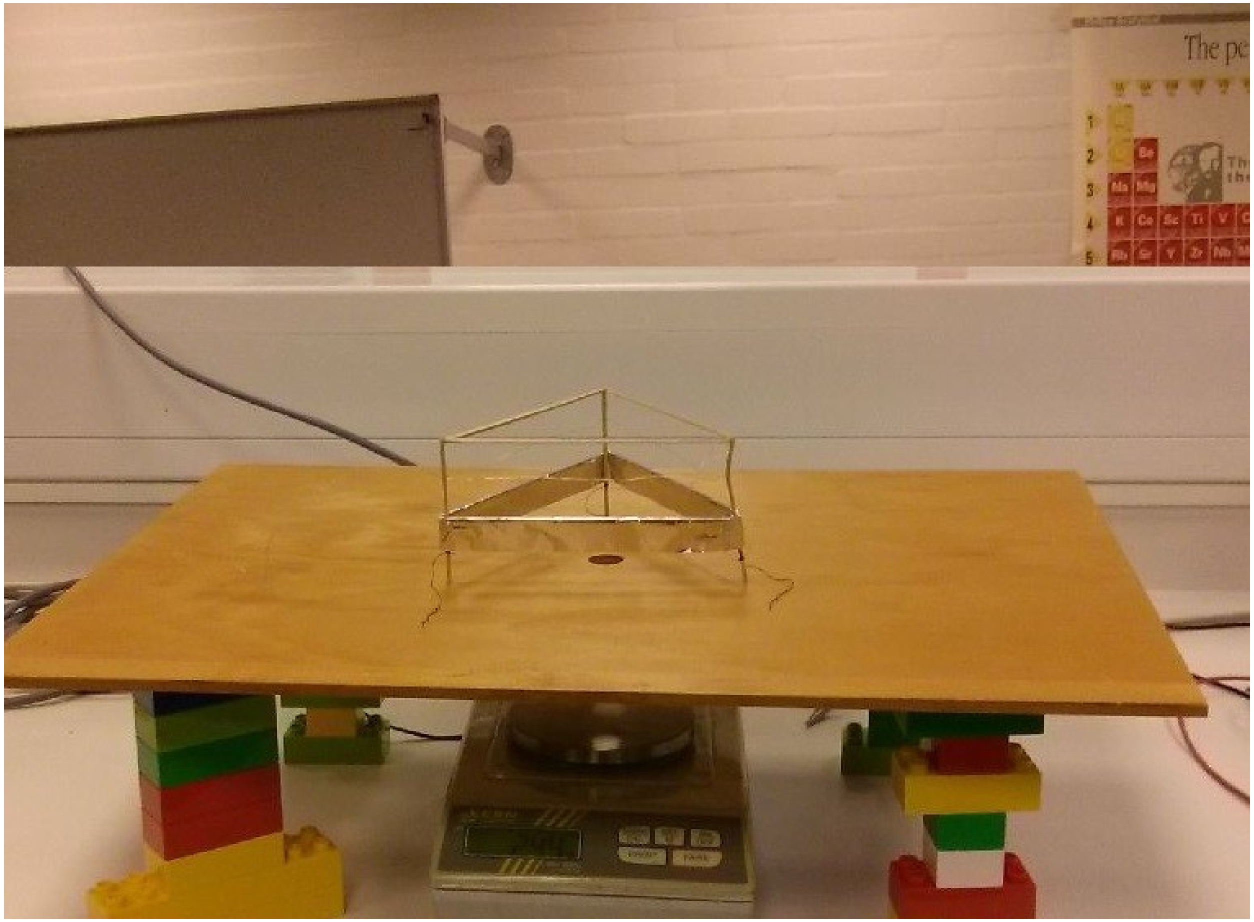
Optimization factors

- Wire diameter
- Air gap
- Aluminum foil width
- Size of lifter
- Type of lifter

Experimental setup

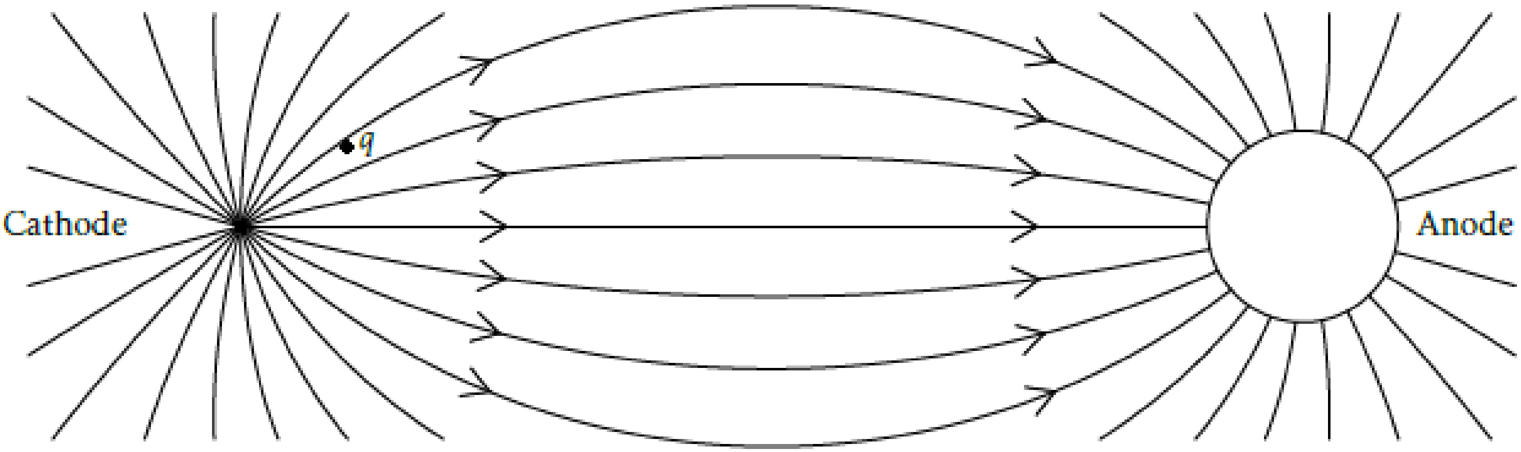


Experimental setup 2

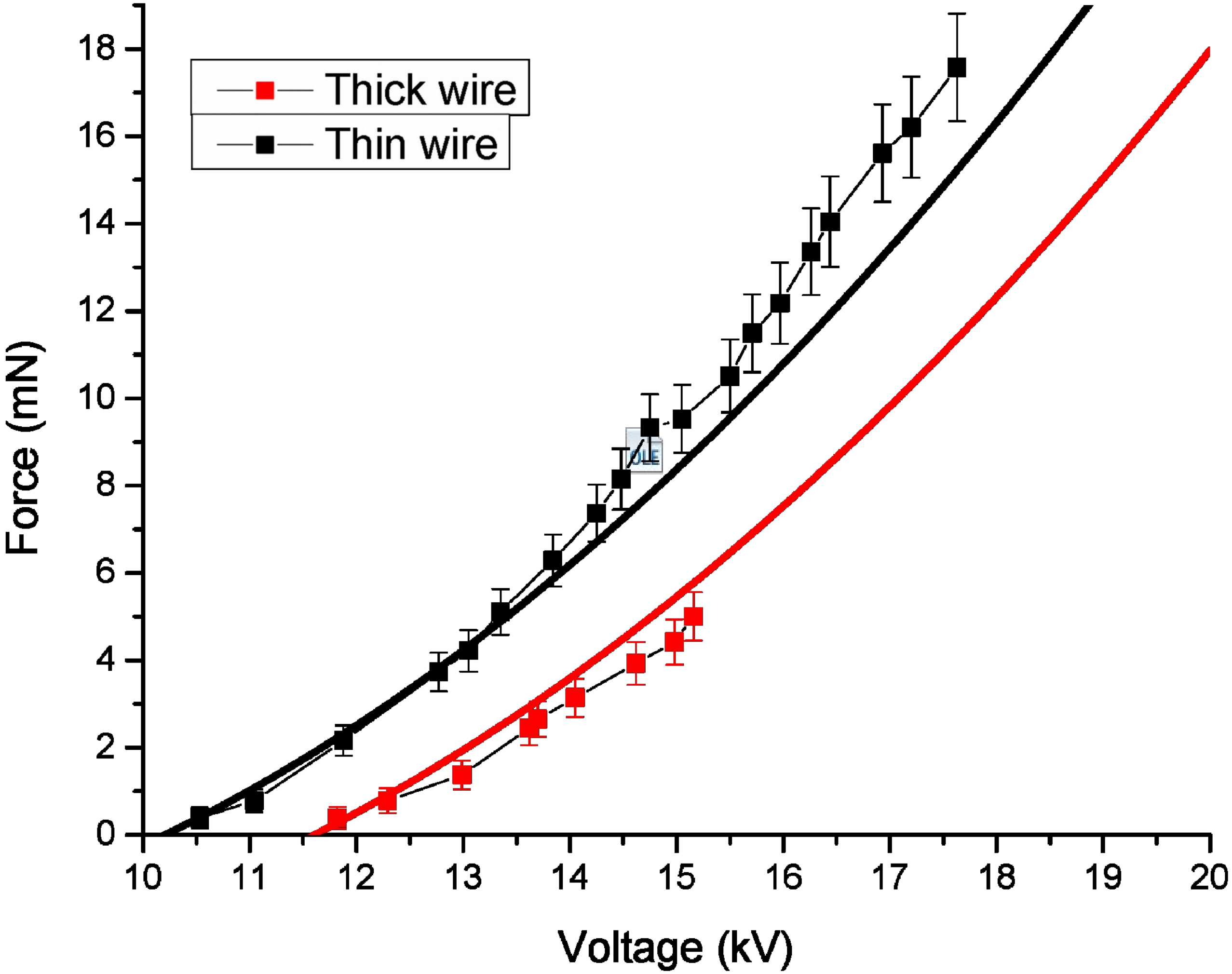


Optimization: Wire size

- Two different radii: 0.173 mm and 0.107 mm



Experimental results: Wire size



Optimization: Air gap

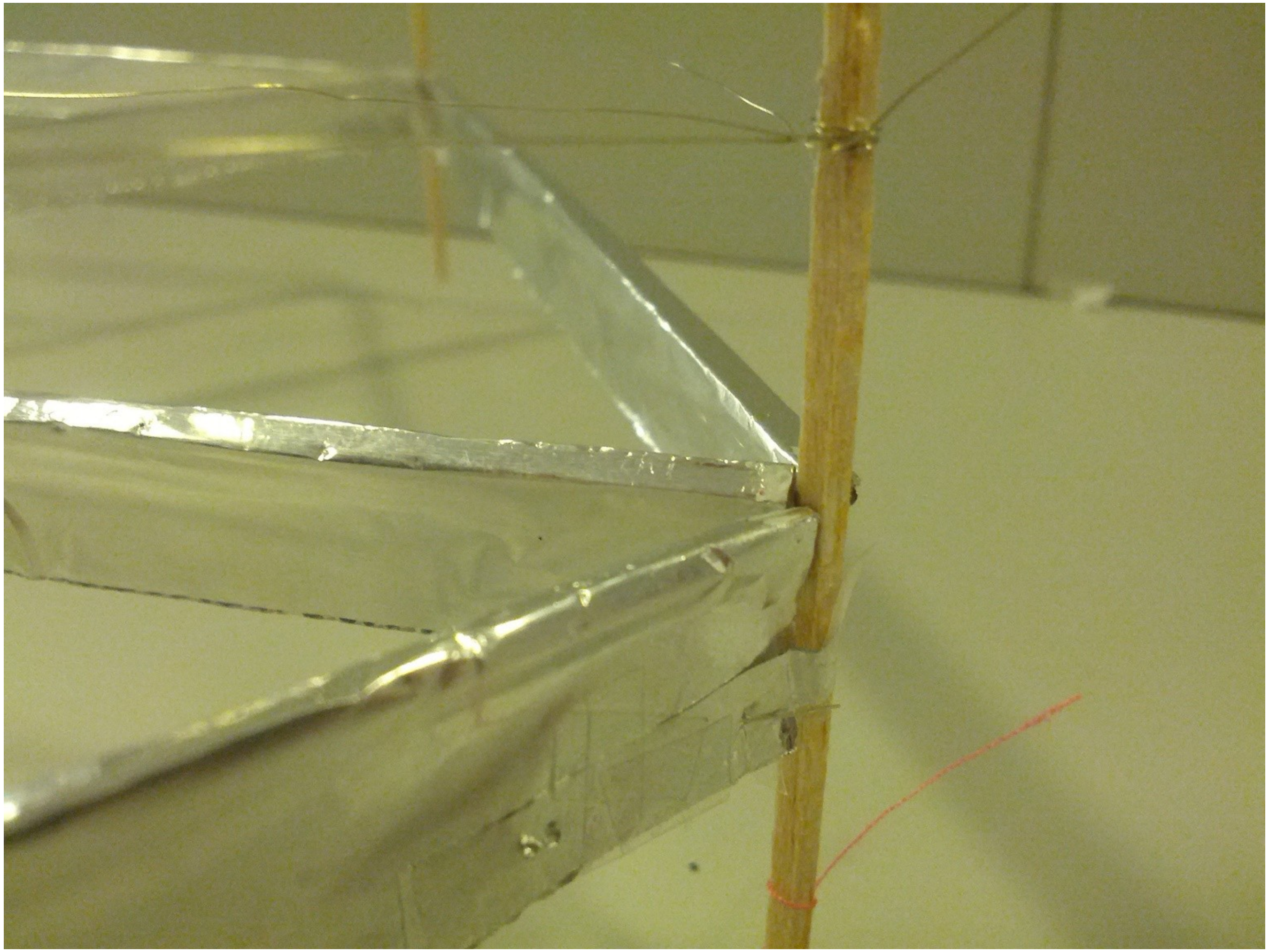
- $$CIV \propto \ln(d)$$
  $$F \propto \frac{1}{d \cdot \ln(x) + d^2}$$

- The smaller the air gap, the smaller the CIV, and the greater the generated lift.

- However, if the distance is too small, breakdown of air will occur, which shortcircuts the setup.

- Breakdown of dry air is $3 \cdot 10^6$ V/m = 30 kV/cm.

Optimization: Air gap



Optimization: Air gap

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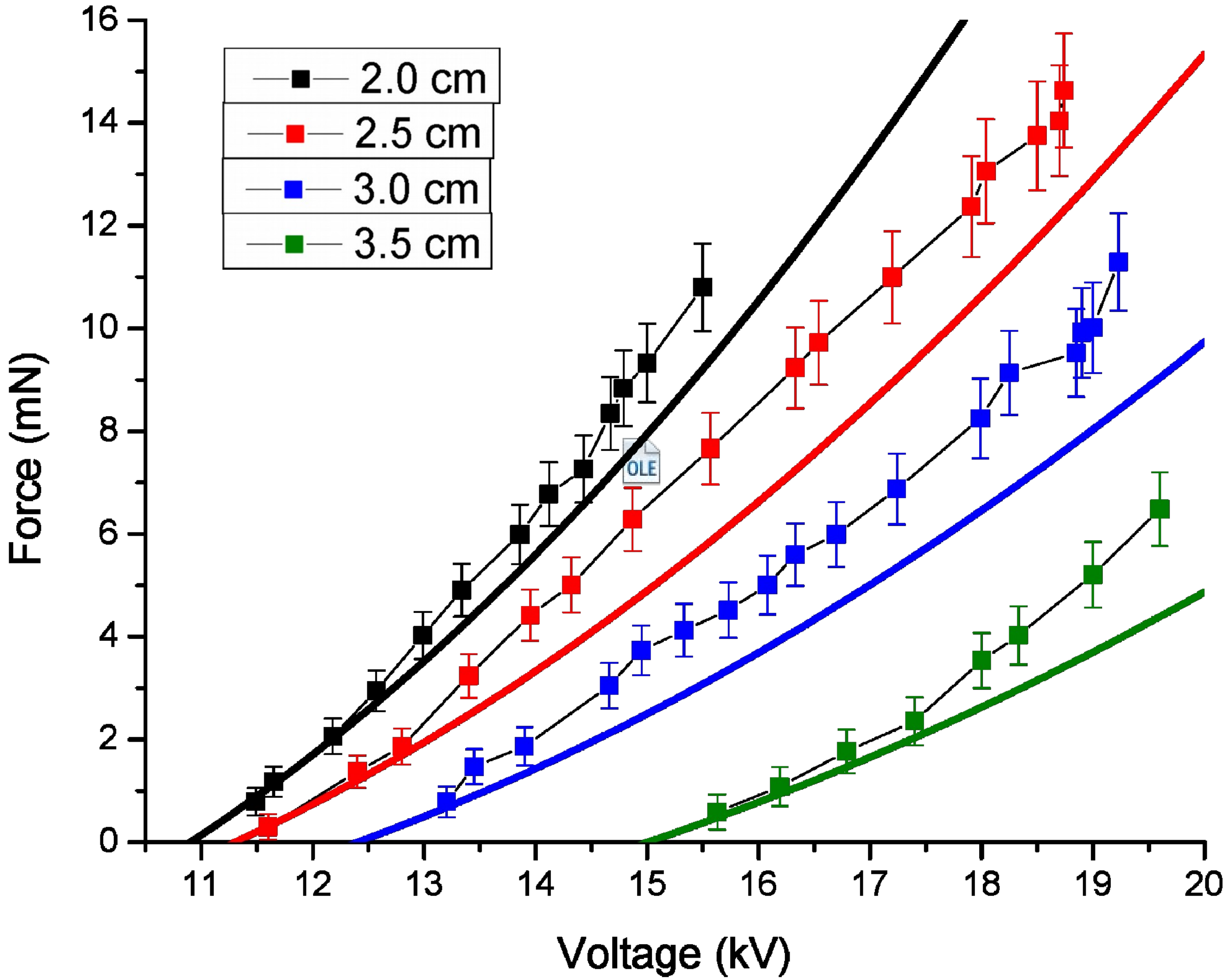
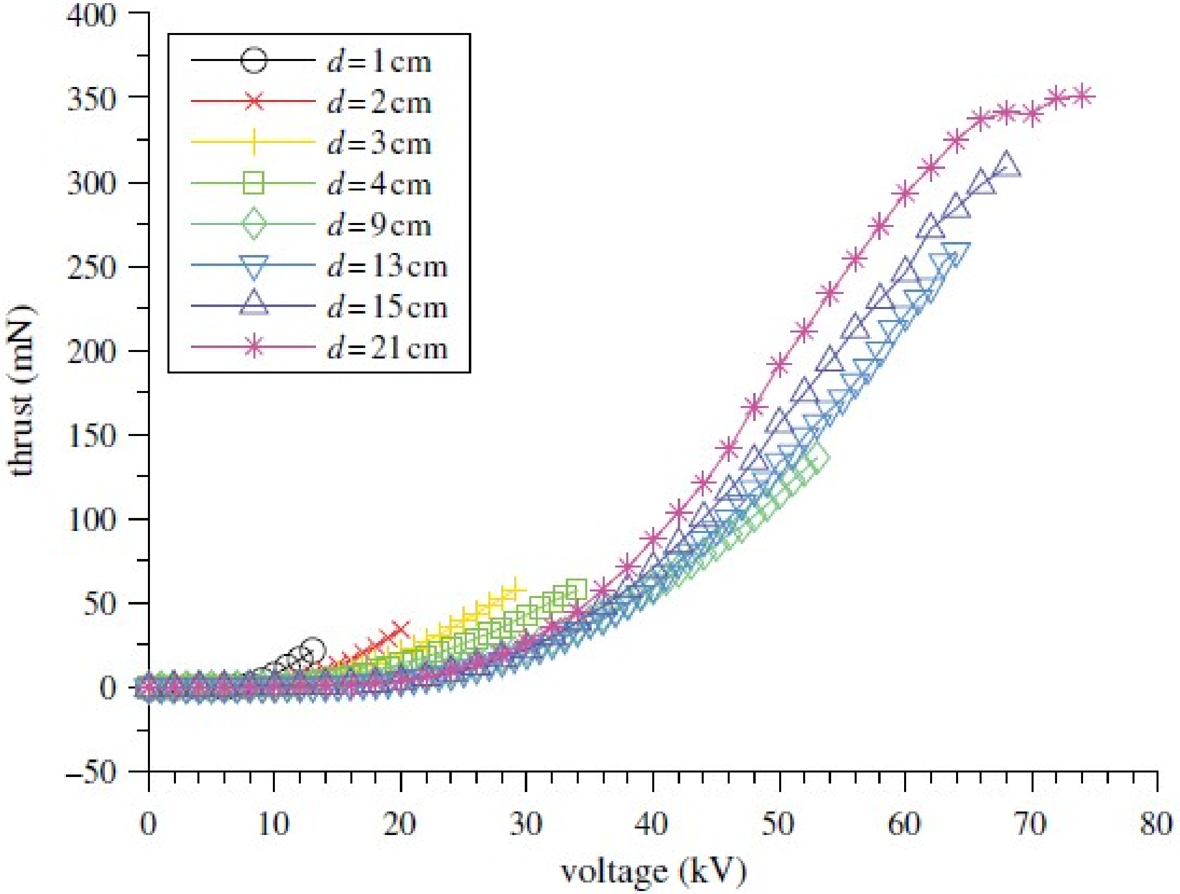


Optimization: Air gap



Graph taken from [1] Kento Masuyama, Steven Barrett, ”On the performance of electrohydrodynamic propulsion”, Proc. R. Soc. A., 2013, pg. 469.

Optimization: Aluminum foil width

- $$f_d = \frac{1}{2} \cdot \rho \cdot v^2 \cdot C_d \cdot A$$

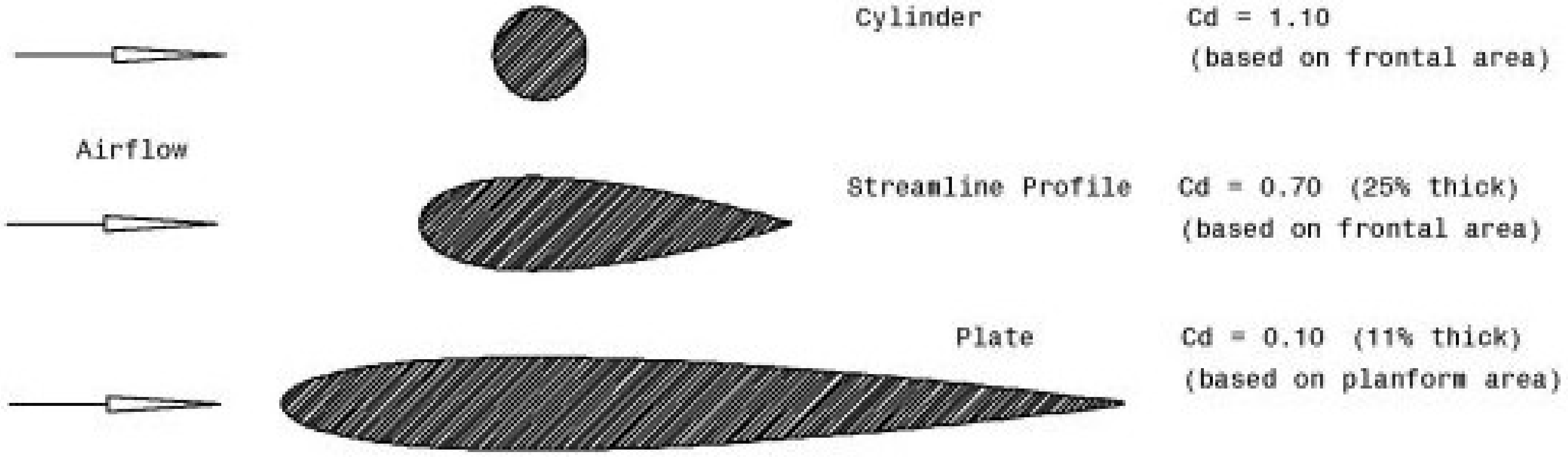
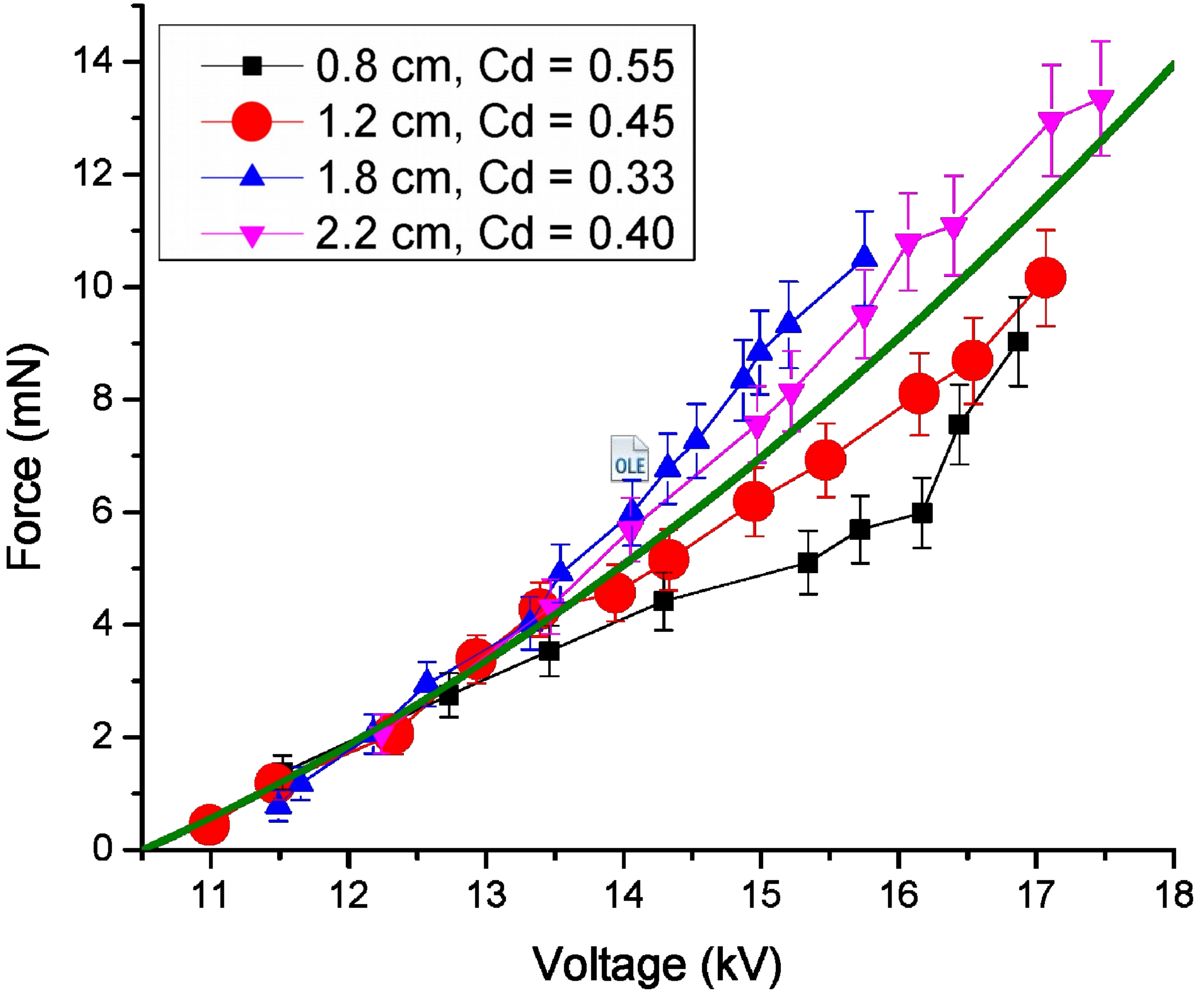


Figure from: “Estimation of The Drag of a Roof Mounted Antenna (Ford AU Falcon)”, http://www.virtualv8.com/freport.htm

Optimization: Aluminum foil width



Optimization: Size of lifter

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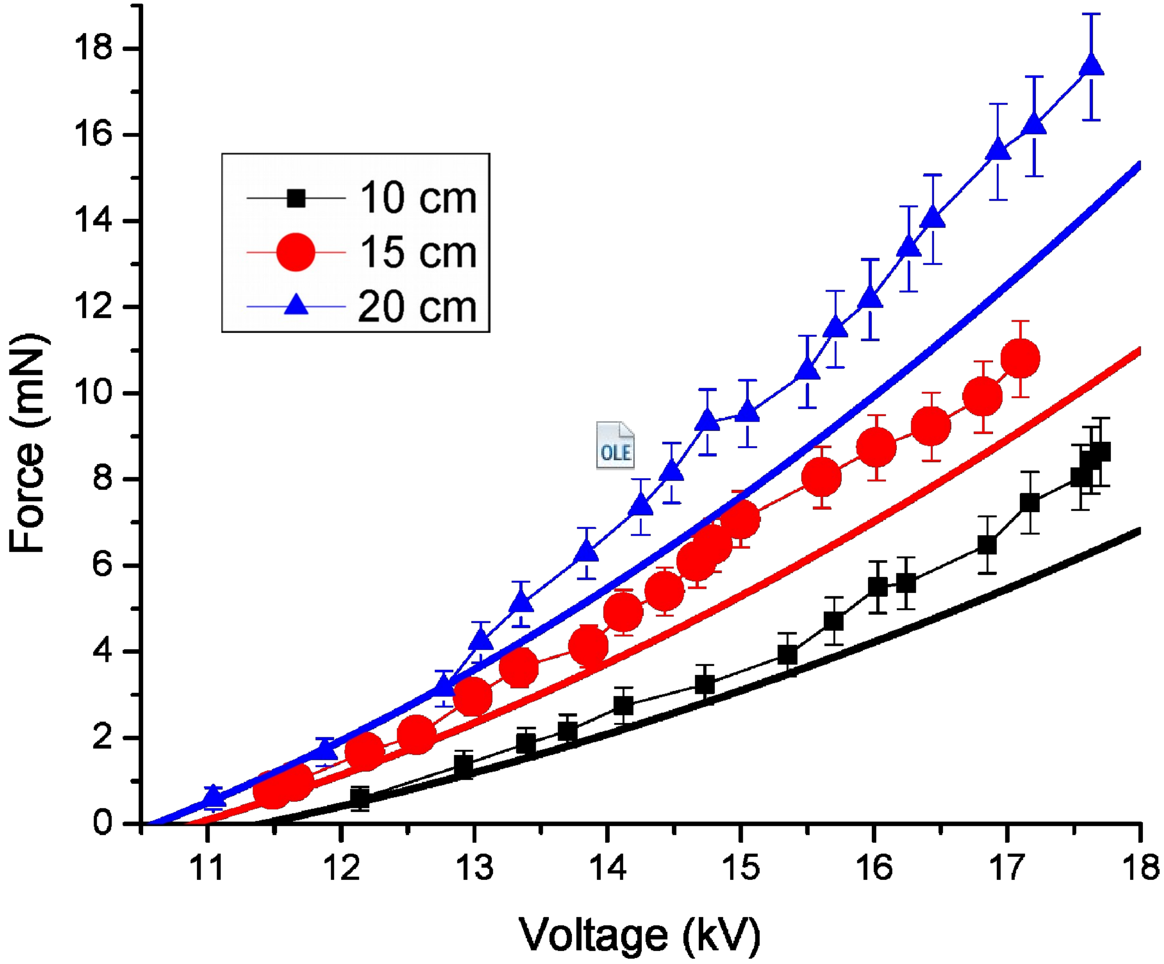
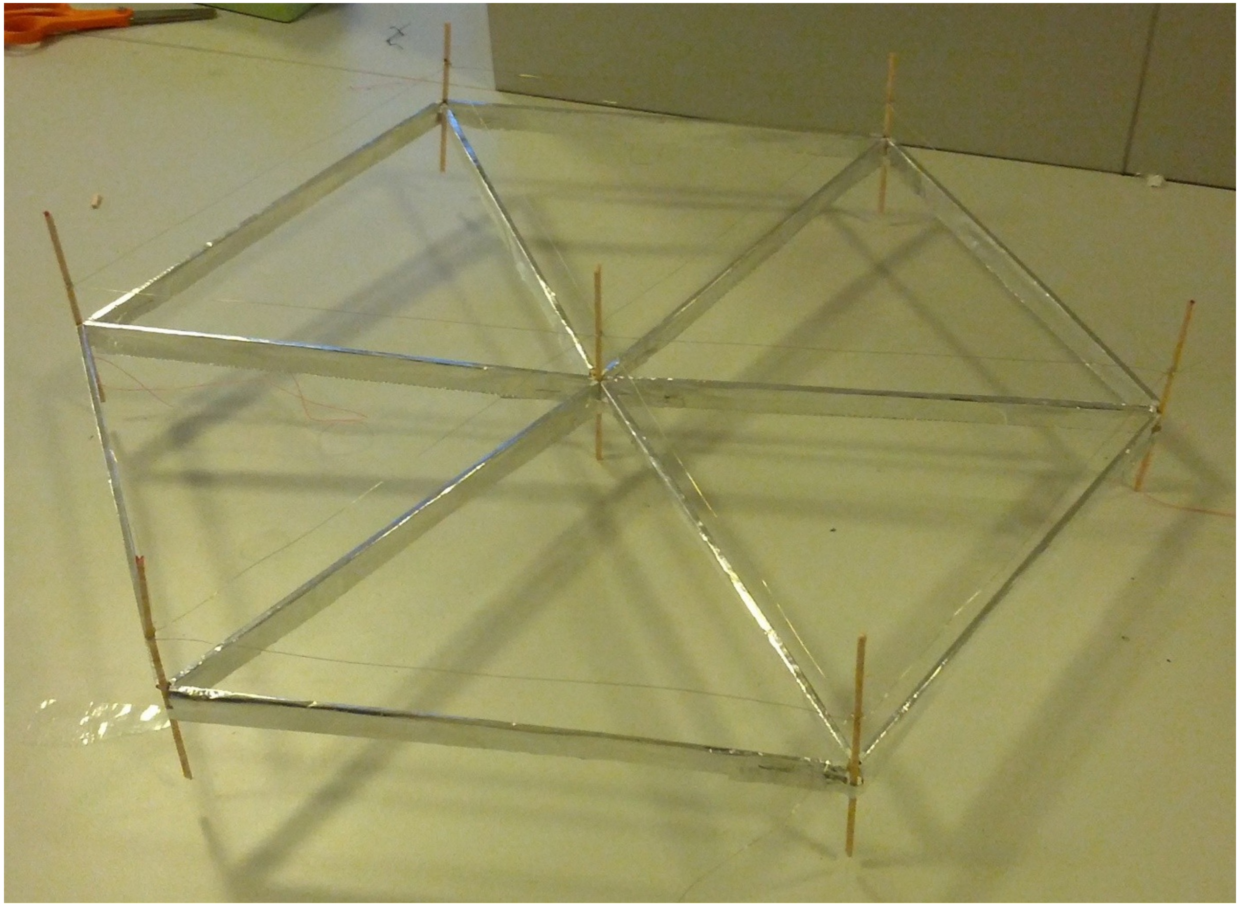
$$F \propto L$$

- A bigger lifter will generate more thrust, however, it will also weigh more.
- Eventually the lifter will need extra support pillers, to prevent the coronating wire from being too slack.

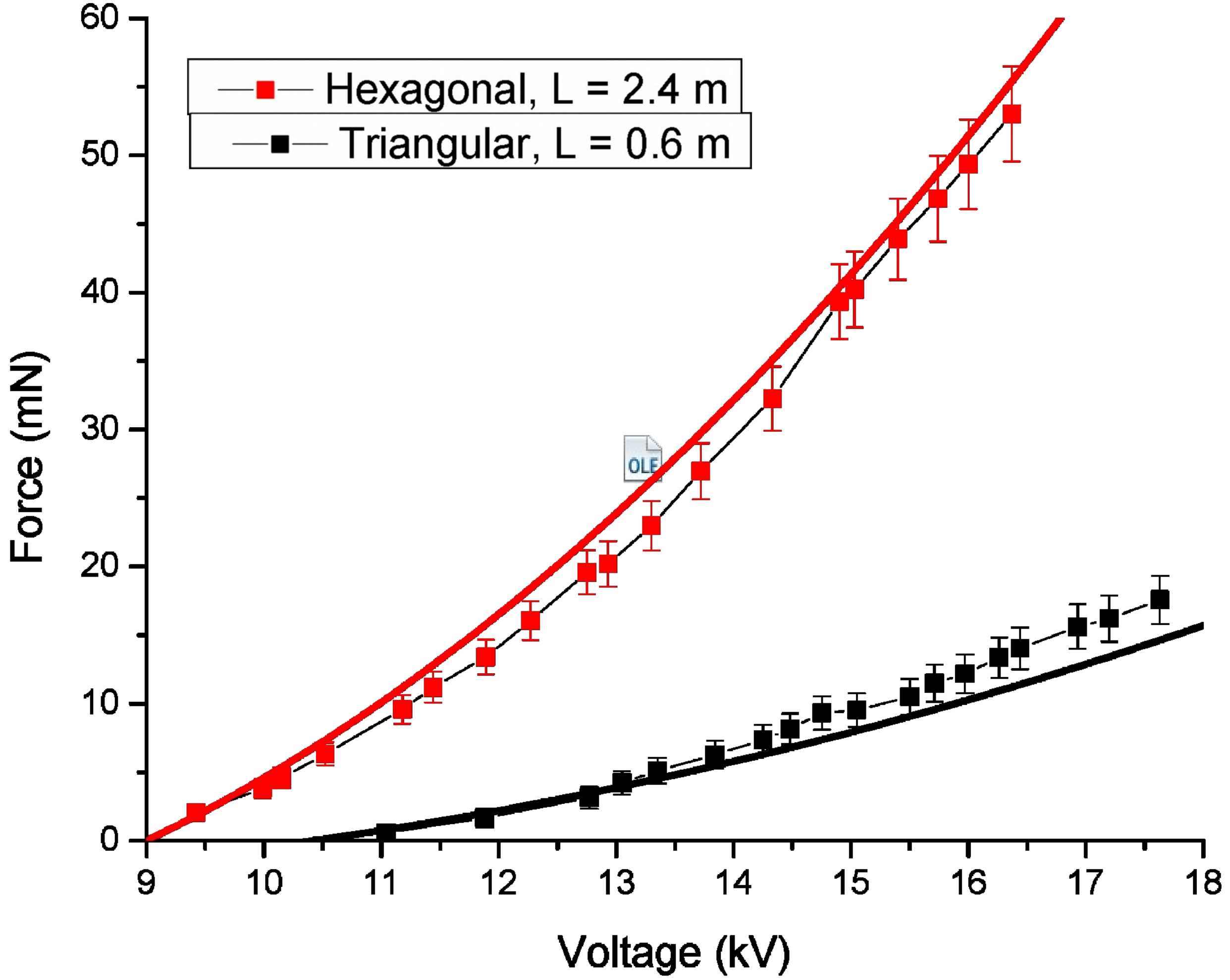
Optimization: Size of lifter



Optimization: Types of lifter



Optimization: Types of lifter



Conclusion

- Thinnest possible coronating wire.
- Air gap depends on voltage input.
- As much length as possible, without extra support pillars.
- Longer, straight, aluminum foil width.
- Hexagonal types are more efficient than triangular.

$$F \propto \frac{V\left[V-\left(1+\frac{1}{\sqrt{r}}\right)\ln\left(\frac{d}{r}\right)\right]L}{d\ln(x)+d^{2}}$$

- Ideal lifter is circular, but that is very hard to build.

Thank you for your attention.

References

- [1] Kento Masuyama, Steven Barrett, ”On the performance of electrohydrodynamic propulsion”, Proc. R. Soc. A., 2013, pg. 469.
- [2] Gene Cooperman, ”A new current-voltage relation for duct precipitators valid for low and high curent densities”, Computer and science faculty publications, 1981, paper 12.
- [3] P. Cooperman, ”A theory for space-charge-limited currents with aplication to electrical application”, AIEE Trans., Vol 79, no. 47, 1960.
- [4] Clemens Wan, ”Electro-Hydrodynamic (EHD) thruster analysis and optimization”, The cooper union for the advancement of science an art, 2009.
- [5] Nick Andersen, Kasper Larsen, ”The electrostatic levitation unit”, Technical university of Denmark, FYS, special project, 10064, 2008.
- [6] A. I. Tsaturyan, ”Current-voltage characteristic of a corona discharge in a dispersive medium”, UDC, 651.359.482
- [7] E. Barsoukov, ”Lifter theory explained”, JNL Labs, Apr. 30, 2002.
- [8] J. S. Townsend, ”Handbuch der Radiologie”, Leipzig, Germany, vol. 1, 1920.

Appendix A: CIV range

-

- Asymmetry of cathode and anode. We can only calculate a range for the CIV.

$$CIV = E_0 \cdot \delta \cdot \left(1 + \frac{\gamma}{\sqrt{\delta \cdot r}}\right) \cdot m_v \cdot r \cdot \ln\left(\frac{d}{r}\right)$$

- For r seperately insert $r_{cathode}$ and r_{anode} and calculate range.

Appendix A: CIV range

- Asymmetry of cathode and anode. We can only calculate a range for the CIV.

$$CIV = E_0 \cdot \delta \cdot \left(1 + \frac{\gamma}{\sqrt{\delta \cdot r}}\right) \cdot m_v \cdot r \cdot \ln\left(\frac{d}{r}\right)$$

- For $d = 2\ \mathrm{cm}$
  - $r_{corona} = 0.173\ \mathrm{mm}$ $8{,}095\ \mathrm{V} < CIV < 17{,}537\ \mathrm{V}$
  - $r_{corona} = 0.107\ \mathrm{mm}$ $6{,}550\ \mathrm{V} < CIV < 17{,}537\ \mathrm{V}$

Appendix A: Townsend equation

-
- Slightly changed Poisson equation:

$$\nabla \cdot E = \frac{1}{\varepsilon_0} \cdot (\rho_i + \rho_\sigma)$$

- ρ_i is the ionic space charge density
- ρ_σ is the space charge density of dispersive phase.

$$\rho_\sigma = \sigma \cdot \varepsilon_0 \cdot E$$

- Cylindrical coor.: $\frac{1}{r} \cdot \frac{d(rE)}{dr} = \frac{\rho_i}{\varepsilon_0} + \sigma \cdot E$
- Solution:

$$E = E_{0\sigma} \cdot r_0 \cdot \frac{e^{\sigma r}}{r} + \frac{\rho_i}{\varepsilon_0 \sigma^2} \left(\frac{e^{\sigma r}}{r} - \frac{1}{r} - \sigma \right)$$

- $E_{0\sigma}$ is the electric field intensity at surface of coronating electrode

Appendix A: Townsend equation

- To find applied voltage, integrate E:

$$V = \int_{r_0}^{R} E\,dr = E_{0\sigma} r_0 \ln\left(\frac{R}{r_0}\right)\left[1 + \frac{\sum_{n=1}^{\infty} \frac{(R\sigma)^n}{n \cdot n!}}{\ln\left(\frac{R}{r_0}\right)}\right] + \frac{\rho_i}{\varepsilon_0} R^2 \sum_{n=2}^{\infty} \frac{(R\sigma)^{n-2}}{n \cdot n!}$$

- Townsend assumptions of neglible field distortion due to ρ_i and the constant ρ_σ along force lines. Medium is assumed isotropic.

$$E_{0\sigma} = E_0 = \frac{V_0}{r_0 \ln\left(\frac{R}{r_0}\right)} \qquad E_R = \frac{V}{R \ln\left(\frac{R}{r_0}\right)} \qquad \rho_i = \frac{i}{2\pi R \kappa E_r} = \frac{i}{2\pi \kappa V} \ln\left(\frac{R}{r_0}\right)$$

- Where κ is the ion mobility and i the current density

Appendix A: Townsend equation

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  - Insert in V:

$$V = V_0\left[1+\frac{\sum_{n=1}^{\infty}\frac{(R\sigma)^n}{n\cdot n!}}{\ln\left(\frac{R}{r_0}\right)}\right]+\frac{iR^2}{V}\,\frac{\ln\left(\frac{R}{r_0}\right)}{2\pi\kappa\varepsilon_0}\sum_{n=2}^{\infty}\frac{(R\sigma)^{n-2}}{n\cdot n!}$$

  - Isolate current density

$$i=\frac{2\pi\kappa\varepsilon_0}{R^2\ln\left(\frac{R}{r_0}\right)}\cdot\frac{1}{\sum_{n=2}^{\infty}\frac{(R\sigma)^{n-2}}{n\cdot n!}}\cdot V\left[V-V_0\left(1+\frac{\sum_{n=1}^{\infty}\frac{(R\sigma)^n}{n\cdot n!}}{\ln\left(\frac{R}{r_o}\right)}\right)\right]$$

  - Which gives the Townsend equation where $\sigma = 0$

$$i=\frac{8\pi\kappa\varepsilon_0}{R^2\ln\left(\frac{R}{r_0}\right)}V(V-V_0)$$

Appendix A: Current-voltage characteristic

- Townsend equation for current-voltage characteristic in coaxial cylindrical system:
- $$I = \frac{8\cdot\pi\cdot\varepsilon_0\cdot\kappa}{d^2\cdot\ln\left(\frac{d}{r}\right)}\cdot V(V - CIV)$$
- Cooperman formats this to fit duct precipitators:
- $$I = \frac{4\cdot\pi\cdot\varepsilon_0\cdot\kappa}{d^2\cdot\ln\left(\frac{fgeo}{r}\right)}\cdot V(V - CIV)$$
- Where f_{geo} is a characteristic length of a particular electrode configuration

Appendix A: Current-voltage characteristic

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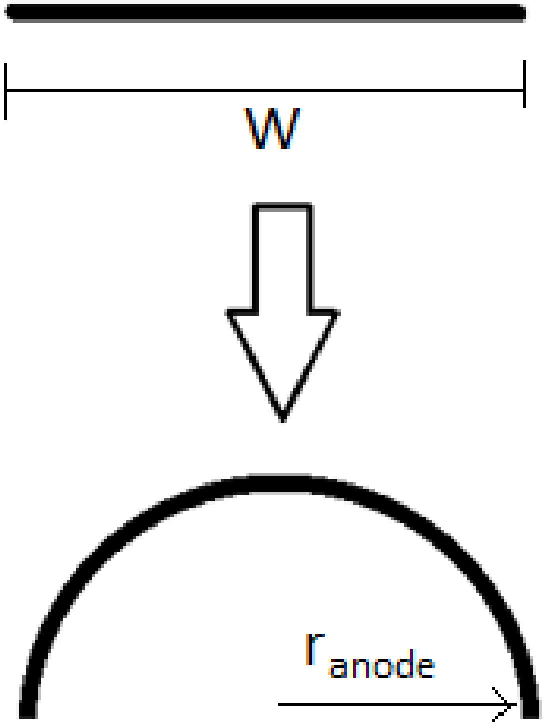
  - Barsoukov fits this characteristic length to a wire-plate capacitor setup:

- $I = \frac{2 \cdot \pi \cdot \varepsilon_0 \cdot \kappa \cdot L}{d^2 \cdot \ln\left(\frac{fgeo}{r}\right)} \cdot V(V - CIV)$

- $f_{geo} = \frac{4 \cdot d}{\pi}$ if $\frac{2 \cdot d}{W} \leq 0.6$
- $f_{geo} = \frac{W \cdot \pi}{2} e^{\frac{\pi \cdot d}{W}}$ if $\frac{2 \cdot d}{W} \geq 2.0$

Appendix A: Current-voltage characteristic

- A formatted Townsend equation is used for wire-plate capacitor setup.
- Since our anode is not a plate, we estimate W to be the top half of a circle:



$$W = \frac{1}{2}\cdot 2\cdot\pi\cdot r_{an}$$

$$I = \frac{2\cdot\pi\cdot\varepsilon_0\cdot\kappa\cdot L}{d^2\cdot\ln\left(\frac{\pi^2\cdot r}{2\cdot r_{an}}\cdot e^{d/r_{an}}\right)}\cdot V(V-CIV)$$

Appendix B: Uncertainties

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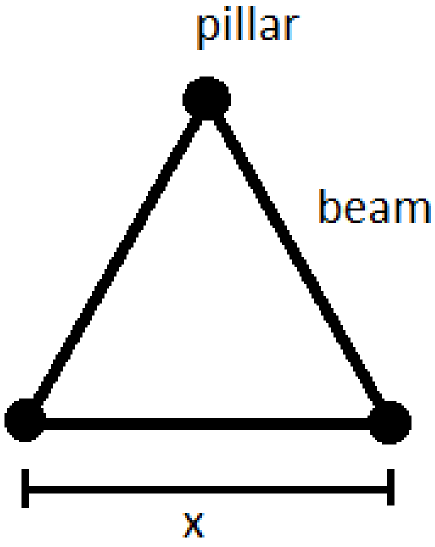
- To calculate uncertainties in our equation for lift:

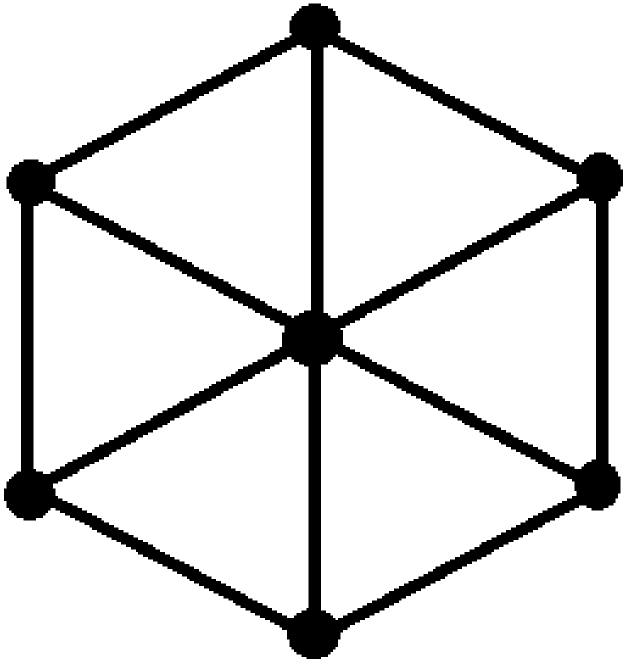
$$\delta F = \sqrt{\left(\frac{\partial F}{\partial x}\delta x\right)^2 + \left(\frac{\partial F}{\partial y}\delta y\right)^2 + \cdots + \left(\frac{\partial F}{\partial z}\delta z\right)^2}$$

- Uncertainties stem from: Voltmeter, weight, length of electrodes, and distance between electrodes.

- The biggest uncertainty, smoothness of collector electrode, cannot correctly be measured, and is therefore not included.

Appendix C: Optimization of types of lifter





L=3x

M=3*beam+3*pillar

L=12x

M=12*beam+7*pillar

Appendix D: Drag and momentum transfer

$$\Delta P_L = \sum_N m_0 \cdot V_{0,N}$$

$$F_L = \frac{dP_L}{dt} = \frac{\beta \cdot m_0 \cdot N_{0,atm} < V_0 >}{\Delta t}$$

$$F_L = \frac{\beta \cdot m_0 \cdot n_{0,atm} \cdot VOL \cdot (< V_0 >)^2}{l}$$

$$F_L = \beta \cdot m_0 \cdot n_{0,atm} \cdot A_{eff} \cdot (< V_0 >)^2 \cdot \cos(< a >)^2$$

Optimization: Types of lifter